Robust Fuzzy Filter Design for a Class of Nonlinear Discrete-Time Uncertain Systems

Chung-Shi Tseng, Bore-Kuen Lee, and Yen-Fang Li

Abstract

This paper studies fuzzy filtering design for a class of nonlinear discrete-time uncertain stochastic systems. First, the Takagi and Sugeno fuzzy model is proposed to approximate a nonlinear discrete-time uncertain stochastic system. Next, based on the fuzzy model, the fuzzy estimation for nonlinear discrete-time uncertain stochastic systems is studied. Using a suboptimal approach, the fuzzy estimation problem for nonlinear discrete-time uncertain stochastic systems is characterized in terms of an eigenvalue problem (EVP) by minimizing the upper bound on the variance of the estimation error. The EVP can be efficiently solved using convex optimization techniques.

Keywords: fuzzy filtering, T-S fuzzy model, nonlinear uncertain stochastic systems, discrete-time, and EVP.

1. Introduction

In general, it is a difficult work to design an efficient filter for nonlinear uncertain stochastic systems. In many cases, systems under consideration are inherently uncertain and nonlinear. Conventionally, a nonlinear estimation algorithm, known as the extended Kalman filter, has been proposed for state estimation by a linearization of the nonlinear systems around the present estimate through an application of linear filter theory [1], [2]. The design of an extended Kalman filter relies on having an exact dynamic model of the system under consideration in order to provide a linearized model around the present estimate [3]. Since the uncertainties are not considered in the design of the extended Kalman filter, the performance of the extended Kalman filter may not be satisfactory in the presence of uncertainties. It is well known that the performance of the extended Kalman filter may deteriorate significantly when the design contains relatively small modeling errors [3]. For the uncertain linear systems, many recent works [3], [4], [5] have dealt with the problem of robust filter design for all admissible uncertainties by minimizing the upper bound on the variance of estimation error, where the uncertainties are parameterized in terms of a norm-bounded parameter matrix. However, few works have studied the estimation problem for nonlinear uncertain stochastic systems [6], [7]. The solution of the filtering problem for the nonlinear stochastic systems is governed by a certain Hamil-ton-Jocobi-Bellman equation (or inequality). In general, it is difficult to solve the Hamilton-Jocobi-Bellman equation (or inequality) efficiently. In this paper, the robust filtering design problem is extended from linear uncertain stochastic systems to a class of nonlinear uncertain stochastic systems using fuzzy approach.

Recently, there have been many applications of fuzzy systems theory in various fields, for example control systems, communication systems and signal processing. In most of these applications, the fuzzy systems were thought of as universal approximators for any nonlinear system. The Takagi and Sugeno fuzzy model [8] which has been proved to be a very good representation for a certain class of nonlinear dynamic systems was extensively studied in control systems [9], [10]. In this study, a Takagi and Sugeno fuzzy model is proposed to approximate a class of nonlinear uncertain stochastic systems. Based on the fuzzy model and using a suboptimal approach, the robust fuzzy filtering design problem is characterized in terms of minimizing the upper bound on the variance of the estimation error subject to some forms of linear matrix inequalities (LMIs). The problem of minimizing the upper bound on the variance of the estimation error, subject to some LMIs, is a standard eigenvalue problem (EVP) which can be efficiently solved by the convex optimization algorithm [11].

The primary contribution of this paper is that it extends the robust filtering design problem from linear uncertain stochastic systems to a class of nonlinear uncertain stochastic systems using fuzzy approach. Furthermore, a systematic design procedure is developed by convex optimization algorithm which can be solved by the LMI optimization toolbox in Matlab. This study is modified from the conference paper [12].

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The paper is organized as follows: A robust discrete-time suboptimal fuzzy filtering design for a class of nonlinear uncertain systems is presented in Section 2. In Section 3, a simulation example is provided to demonstrate the design procedure. Finally, concluding remarks are made in Section 4. In what follows,

$$\begin{bmatrix} M_{11} & * \\ M_{12}^T & M_{22} \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{12}^T & M_{22} \end{bmatrix}$$

throughout this paper.

2. Fuzzy Filtering for Nonlinear Discrete-Time Uncertain Systems

A fuzzy dynamic model has been proposed by Takagi and Sugeno [8] to represent locally linear input/output relations for nonlinear system. This fuzzy dynamic model is described by fuzzy If-Then rules and will be employed here to deal with the filtering design problem of a class of nonlinear discrete-time uncertain system. The ith rule of the fuzzy model for a nonlinear discrete-time uncertain system is proposed as the following form:

Rule i:

If $z_1(\kappa)$ is F_{i1} and $z_g(\kappa)$ is F_{ig}

Then
$$x(\kappa+1) = (A_i + U_{1i}\tilde{A}(\kappa)V_{1i})x(\kappa) + \omega(\kappa)$$
$$y(\kappa) = (C_i + U_{2i}\tilde{C}(\kappa)V_{2i})x(\kappa) + v(\kappa)$$
$$s(\kappa) = Gx(\kappa) \tag{1}$$

for i=1,2,…,L where $x(\kappa) \in R^{n\times 1}$ denotes the state vector; $y(\kappa) \in R^{m\times 1}$ and $s(\kappa) \in R^{q\times 1}$ denote measured output and linear combination of the state variables to be estimated, respectively; $\omega(\kappa) \in R^{n\times 1}$ and $v(\kappa) \in R^{m\times 1}$ are assumed to be uncorrelated, zero-mean, white noise signals; $G \in R^{q\times n}$ is constant matrix; F_{ij} is the fuzzy set $A_i \in R^{n\times n}$ and $C_i \in R^{m\times n}$ are known constant matrices;

U_{1i}, U_{2i}, V_{1i} and V_{2i} are known constant matrices with appropriate dimensions; $\tilde{A}(\kappa)$ and $\tilde{C}(\kappa)$ with appropriate dimensions are uncertain time-varying matrix functions satisfying $\tilde{A}(\kappa)\tilde{A}^T(\kappa) \le I$ and $\tilde{C}(\kappa)\tilde{C}^T(\kappa) \le I$, respectively; L is the number of If-Then rules; and $z_1(\kappa), \ldots, z_g(\kappa)$ are the premise variables.

The final output of the fuzzy system is inferred as follows:

$$x(\kappa+1) = \sum_{i=1}^{L} h_i(z(\kappa))[(A_i + U_{1i}\tilde{A}(\kappa)V_{1i})x(\kappa) + \omega(\kappa)] \tag{2}$$

and

$$y(\kappa) = \sum_{i=1}^{L} hi(z(\kappa))[C_i + U_{2i}\tilde{C}(\kappa)V_{2i})x(\kappa) + v(\kappa)], \tag{3}$$

where

$$h_i(z(\kappa)) = \frac{\mu_i(z(\kappa))}{\sum_{i=1}^{L}\mu_i(z(\kappa))},\ \mu_i(z(\kappa)) = \prod_{j=1}^{g} F_{ij}(z_j(\kappa)), \tag{4}$$
$$z(\kappa) = [z_1(\kappa), z_2(\kappa), \cdots, z_g(\kappa)]$$

and $F_{ij}(z_j(\kappa))$ is the grade of membership of $z_j(\kappa)$ in F_{ij} [13].

It is assumed that

$\mu_i(z(\kappa)) \ge 0$, and $\sum_{i=1}^{L}\mu_i(z(\kappa)) > 0$.

Therefore, we get

$$h_i(z(\kappa)) \ge 0 \text{ and } \sum_{i=1}^{L} h_i(z(\kappa)) = 1 \tag{5}$$

Based on the fuzzy model (1), the following fuzzy estimator is proposed to deal with the state estimation for the nonlinear discrete-time uncertain systems

Estimation Rule i:

If $\hat{z}_1(\kappa)$ is F_{i1} …and $\hat{z}_g(\kappa)$ is F_{ig} (6)

Then $\hat{x}_1(\kappa+1) = A_i\hat{x}(\kappa) + K_i(\ y(\kappa) - c_i\hat{x}(\kappa)\ )$

where K_i is the fuzzy estimator gain for the ith estimation rule.

The overall fuzzy estimator is written as

$$\hat{x}(\kappa+1) = \sum_{i=1}^{L} h_i(\hat{z}(\kappa))\sum_{j=1}^{L} h_i(z(\kappa))[(A_i - k_iC_i)\hat{x}(\kappa) + K_i(C_j + U_{2j}\tilde{C}(\kappa)V_{2j})x(\kappa) + K_iv(\kappa)] \tag{7}$$

Then, the augmented system is defined as the following form

$$\hat{x}(\kappa+1) = \sum_{i=1}^{L} h_i(\hat{z}(\kappa))\sum_{j=1}^{L} h_i(z(\kappa))[(\bar{A}_{ij} + \bar{U}_{ij}\tilde{\Lambda}(\kappa)\bar{V}_j)\tilde{x}(\kappa) + \Gamma_i\tilde{\varpi}(\kappa)] \tag{8}$$

where

$$\hat{x}(\kappa) = \begin{bmatrix} x(\kappa) \\ \hat{x}(\kappa) \end{bmatrix},\quad \bar{A}_{ij} = \begin{bmatrix} A_j & 0 \\ K_iC_i & (A_i - K_iC_i) \end{bmatrix},$$
$$\bar{U}_{ij}(\kappa) = \begin{bmatrix} U_{1j} & 0 \\ 0 & K_iU_{2j} \end{bmatrix},\quad \bar{V}_j = \begin{bmatrix} V_{1j} & 0 \\ V_{2j} & 0 \end{bmatrix},$$
$$\tilde{\Lambda}(\kappa) = \begin{bmatrix} \tilde{A}(\kappa) & 0 \\ 0 & \tilde{C}(\kappa) \end{bmatrix},\quad \bar{\Gamma}_i = \begin{bmatrix} I & 0 \\ 0 & K_i \end{bmatrix},$$
$$\tilde{\varpi}(\kappa) = \begin{bmatrix} \omega(\kappa) \\ v(\kappa) \end{bmatrix} \tag{9}$$

Remark 1: It is easy to show that $\tilde{\Lambda}(\kappa)\tilde{\Lambda}^T(\kappa) \le I$. ■

Let us denote the estimation error as

$$e(\kappa) = s(\kappa) - \hat{s}(\kappa) = [G \quad -G]\tilde{x}(\kappa) = \bar{G}\tilde{x}(\kappa) \tag{10}$$

We wish to select the estimator gain K_i (for i=1,…,L) such that the upper bound of the following cost function (variance of the estimation error)

$$J = \lim_{\kappa\to\infty} E(e^T(\kappa+1)e(\kappa+1)) = \lim_{\kappa\to\infty} tr(\overline{G}E(\tilde{x}(\kappa+1)\tilde{x}^T(\kappa+1)\overline{G}^T) \quad (11)$$

is minimized.

From (11), the covariance matrix of $\tilde{x}(\kappa+1)$ is obtained as following [12]

$$\lim_{\kappa\to\infty} E(\tilde{x}(\kappa+1)\tilde{x}^T(\kappa+1)) = \lim_{\kappa\to\infty} E\{(\sum_{i=1}^{L}\sum_{j=1}^{L} h_i(\hat{z}(\kappa))h_j(z(\kappa)) \times[(\overline{A}_{ij}+\overline{U}_{ij}\tilde{\Lambda}(\kappa)\overline{V}_j)\tilde{x}(\kappa)+\overline{\Gamma}_i\tilde{\varpi}(\kappa)]) \times(\sum_{l=1}^{L}\sum_{m=1}^{L} h_l(\hat{z}(\kappa))h_m(z(\kappa)) \times[(\overline{A}_{lm}+\overline{U}_{lm}\tilde{\Lambda}(\kappa)\overline{V}_m)\tilde{x}(\kappa)+\overline{\Gamma}_l\tilde{\varpi}(\kappa)])^T\}.$$

By the fact that $M_1^TM_2+M_2^TM_1\le M_1^TM_1+M_2^TM_2$, we obtain

$$\lim_{\kappa\to\infty} E(\tilde{x}(\kappa+1)\tilde{x}^T(\kappa+1)) \le \lim_{\kappa\to\infty} E\{(\sum_{i=1}^{L}\sum_{j=1}^{L} h_i(\hat{z}(\kappa))h_j(z(\kappa)) \times[(\overline{A}_{ij}+\overline{U}_{ij}\tilde{\Lambda}(\kappa)\overline{V}_j)\tilde{x}(\kappa)((\overline{A}_{ij}+\overline{U}_{ij}\tilde{\Lambda}(\kappa)\overline{V}_j)\tilde{x}(\kappa))^T + \overline{\Gamma}_i\tilde{\varpi}(\kappa)(\overline{\Gamma}_i\tilde{\varpi}(\kappa))^T])\}.$$

If $\varepsilon>0$ and $\varepsilon^{-1}I-\overline{V}_j\overline{P}\overline{V}_j^T>0$ [14], we obtain

$$\begin{aligned}\lim_{\kappa\to\infty} E(\tilde{x}(\kappa+1)\tilde{x}^T(\kappa+1)) &\le \sum_{i=1}^{L}\sum_{j=1}^{L}[\overline{A}_{ij}\overline{P}\overline{A}_{ij}^T+\overline{A}_{ij}\overline{P}(\overline{U}_{ij}\tilde{\Lambda}(\kappa)V_j)^T+\overline{\Gamma}_i\overline{R}\overline{\Gamma}_i^T \\ &+\overline{U}_{ij}\tilde{\Lambda}(\kappa)\overline{V}_j\overline{P}\overline{A}_{ij}^T+(\overline{U}_{ij}\tilde{\Lambda}(\kappa)\overline{V}_j)\overline{P}(\overline{U}_{ij}\tilde{\Lambda}(\kappa)\overline{V}_j)^T] \\ &\le \sum_{i=1}^{L}\sum_{j=1}^{L}[\overline{A}_{ij}\overline{P}\overline{A}_{ij}^T+\overline{A}_{ij}\overline{P}\overline{V}_j^T(\varepsilon^{-1}I-\overline{V}_j\overline{P}\overline{V}_j^T)^{-1} \\ &\times\overline{V}_j\overline{P}\overline{V}_{ij}^T+\varepsilon^{-1}\overline{U}_{ij}(\kappa)\overline{U}_{ij}^T+\overline{\Gamma}_i\overline{R}\overline{\Gamma}_i^T],\end{aligned} \quad (12)$$

where constant matrices

$$\overline{P}=\lim_{\kappa\to\infty}E(\tilde{x}(\kappa)\tilde{x}^T(\kappa))\ge 0$$

and

$$\overline{R}=\lim_{\kappa\to\infty}E[\tilde{\varpi}(\kappa)\tilde{\varpi}(\kappa)^T]=diag\,(\overline{R}_w,\overline{R}_v)>0$$

Remark 2: The assumption of $\overline{R}>0$ is required in this study. ■

From (11) and (12), we obtain

$$J=\lim_{\kappa\to\infty}E(e^T(\kappa+1)e(\kappa+1)) \le L^2tr\{\overline{G}\overline{P}\overline{G}^T\}+tr\{\overline{G}\sum_{i=1}^{L}\sum_{j=1}^{L}[\Psi_{ij}(\overline{P})]\overline{G}^T\} \quad (13)$$

where

$$\Psi_{ij}(\overline{P})=\overline{A}_{ij}\overline{P}\overline{A}_{ij}^T+\overline{A}_{ij}\overline{P}\overline{V}_j^T(\varepsilon^{-1}I-\overline{V}_j\overline{P}\overline{V}_j^T)^{-1} \times\overline{V}_j\overline{P}\overline{A}_{ij}^T+\varepsilon^{-1}\overline{U}_{ij}(\kappa)\overline{U}_{ij}^T+\overline{\Gamma}_i\overline{R}\overline{\Gamma}_i^T-\overline{P}$$

If

$$\Psi_{ij}(\overline{p})\le 0 \quad (14)$$

for all i,j=1,…,L, then

$$J\le L^2tr\{\overline{G}\overline{P}\overline{G}^T\} \quad (15)$$

Based on the analysis above, the filtering design is proposed by minimizing the upper bound on the variance of estimation error as the following minimization problem:

$$\min_{\{\overline{P},\kappa_i,\varepsilon\}} tr\{\overline{G}\overline{P}\overline{G}^T\}$$

subject to $\overline{P}=\overline{P}^T\ge 0,\varepsilon>0,$ and $\Psi_{ij}(\overline{P})\le 0$. (16)

To convert (16) into a LMI formulation, the following minimization problem including strict inequalities is considered, if Φ is a solution of the following minimization problem

$$\min_{\{\overline{P},\kappa_i,\varepsilon\}} tr\{\overline{G}\Phi\overline{G}^T\}$$

subject to $\Phi=\Phi^T>0,\varepsilon>0$, and $\Psi_{ij}(\Phi)<0$, (17)

then

$$J\le L^2tr\{\overline{G}\overline{P}\overline{G}^T\}\le L^2tr\{\overline{G}\Phi\overline{G}^T\}. \quad (18)$$

The condition $\Psi_{ij}(\Phi)<0$. in (17), by the Schur complements [11], is equivalent to

$$\begin{bmatrix}-\Phi & * & * & * & * \\ 0 & -\varepsilon^{-1}I & * & * & * \\ A_{ij}^T & \overline{V}_j^T & -\Phi^{-1} & * & * \\ \overline{U}_{ij}^T & 0 & 0 & -\varepsilon I & * \\ \overline{\Gamma}_i^T & 0 & 0 & 0 & -\overline{R}^{-1}\end{bmatrix}<0 \quad (19)$$

Premultiplying and postmultiplying the both sides of the above inequalities by block-diag$\{\overline{Q},\varepsilon I,I,I,I\}$, where $\overline{Q}=\overline{Q}^T=\Phi^{-1}$, the following matrix inequalities can be obtained

$$\begin{bmatrix}-\Phi & * & * & * & * \\ 0 & -\varepsilon I & * & * & * \\ A_{ij}^T\overline{Q} & \varepsilon\overline{V}_j^T & -\overline{Q} & * & * \\ \overline{U}_{ij}^T\overline{Q} & 0 & 0 & -\varepsilon I & * \\ \overline{\Gamma}_i^T\overline{Q} & 0 & 0 & 0 & -\overline{R}^{-1}\end{bmatrix}<0 \quad (20)$$

For the convenience of design, let

$$\overline{Q}=\begin{bmatrix}\overline{Q}_{11} & -\overline{Q}_{22} \\ -\overline{Q}_{22} & \overline{Q}_{22}\end{bmatrix} \quad (21)$$

then (20) is rewritten as following linear matrix inequalities (LMIs)

$$\begin{bmatrix} -\overline{Q}_{11} & * & * & * & * & * & * & * & * & * \\ \overline{Q}_{22} & -\overline{Q}_{22} & * & * & * & * & * & * & * & * \\ 0 & 0 & -\varepsilon I & * & * & * & * & * & * & * \\ 0 & 0 & 0 & -\varepsilon I & * & * & * & * & * & * \\ \Psi_1^T & -\Psi_1^T & \varepsilon V_{1j}^T & 0 & -\overline{Q}_{11} & * & * & * & * & * \\ -\Psi_2^T & \Psi_2^T & \varepsilon V_{2j}^T & 0 & \overline{Q}_{22} & -\overline{Q}_{22} & * & * & * & * \\ \Psi_3^T & -\Psi_4^T & 0 & 0 & 0 & 0 & -\varepsilon I & * & * & * \\ -\Psi_5^T & \Psi_5^T & 0 & 0 & 0 & 0 & 0 & -\varepsilon I & * & * \\ \overline{Q}_{11}^T & -\overline{Q}_{22}^T & 0 & 0 & 0 & 0 & 0 & 0 & -\overline{R}_w^{-1} & * \\ -Y_i^T & Y_i^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\overline{R}_v^{-1} \end{bmatrix} < 0 \qquad (22)$$

where

$\Psi_1 = \overline{Q}_{11}A_j - Y_iC_j, \Psi_2 = \overline{Q}_{22}A_j - Y_iC_j, \Psi_3 = \overline{Q}_{11}U_{1j}$,

$\Psi_4 = \overline{Q}_{22}U_{1j}, \Psi_{5=}Y_iU_{2j}$, and $Y_i = Q_{22}K_i$.

Note that if there exists a symmetric positive-definite matrix $\overline{W} > 0$ such that

$$\overline{G}\,\overline{Q}^{-1}\overline{G}^T < \overline{W} \qquad (23)$$

which is equivalent to the following LMI

$$\begin{bmatrix} -\overline{W} & * \\ \overline{G}^T & -\overline{Q} \end{bmatrix} < 0, \qquad (24)$$

then

$$tr\{\overline{G}\,\overline{Q}^{-1}\overline{G}^T\} < tr(\overline{W}). \qquad (25)$$

Based on the analysis above, a suboptimal filtering design is proposed by minimizing the upper bound on the variance of estimation error as the following eigenvalue problem (EVP):

$$\min_{\{\overline{p},\overline{Q},Y_i,\varepsilon\}} tr(\overline{W})$$

subject to $\overline{W} = \overline{W}^T > 0$, $\overline{Q} = \overline{Q}^T > 0$, $\varepsilon > 0$, (26)

(24) and (22)

Then, we get the following main result.

Theorem 1: If constant symmetric matrices $\overline{W} = \overline{W}^T > 0$ and $\overline{Q} = \overline{Q}^T > 0$ are solutions of the EVP in (26), then the variance of the estimation error is bounded by

$$J = \lim_{\kappa\to\infty} E(e^T(\kappa+1)e(\kappa+1)) < L^2 tr(\overline{W}). \qquad (27)$$

3. Simulation Example

Consider the following nonlinear discrete-time uncertain system.

$$x_1(\kappa+1) = 0.3x_1(\kappa) - 0.2(x_2(\kappa) - x_1^2(\kappa)) + \tilde{f}_1(x(\kappa)) + \omega_1(\kappa)$$

$$x_2(\kappa+1) = 0.2x_1(\kappa) + 0.3(x_2(\kappa) - x_1^2(\kappa)) + \tilde{f}_2(x(\kappa)) + \omega_2(\kappa)$$

$$y(\kappa) = 0.05(x_1(\kappa) + x_2(\kappa)) + 0.1(x_1^2(\kappa) + x_2^2(\kappa)) + \tilde{h}(x(\kappa)) + v(\kappa)$$

$$s(\kappa) = \begin{bmatrix} x_1(\kappa) \\ x_2(\kappa) \end{bmatrix} \qquad (28)$$

where $\tilde{f}_1(x(\kappa))$, $\tilde{f}_2(x(\kappa))$, and $\tilde{h}(x(\kappa))$ are uncertainties which are assumed to be characterized as the following fuzzy rules.

The fuzzy filtering design for nonlinear discrete-time uncertain system in (28) is given by the following steps.

Step 1: Construct the fuzzy model rules as follows:
Rule i: IF x_1 is about F_{i1} and x_2 is about F_{i2} THEN

$$x(\kappa+1) = (A_i + U_{1i}\tilde{A}(\kappa)V_{1i})x(\kappa) + \omega(\kappa)$$

$$y(\kappa) = (C_i + U_{2i}\tilde{C}(\kappa)V_{2i})x(\kappa) + v(\kappa)$$

where A_i, C_i, U_{1i}, V_{1i}, U_{2i}, and V_{2i} (for i = 1,…,9) are shown in Appendix; $\tilde{A}(\kappa) = \begin{bmatrix} N_1(\kappa) & 0 \\ 0 & \sin(0.2\kappa) \end{bmatrix}$, $\tilde{C}(\kappa) = \begin{bmatrix} N_2(\kappa) & 0 \\ 0 & \cos(0.2\kappa) \end{bmatrix}$, $N_j(\kappa)(j=1,2)$ are random sequence with $|N_j(\kappa)| \le 1$, $\forall\kappa$, and $w(\kappa) = [\omega_1(\kappa), \omega_2(\kappa)]^T$,.In this example, for the convenience of simulation, it is assumed that $\omega(\kappa)$ and $v(\kappa)$ are normal distribution noises with zero mean and variance 0.01I and 0.01,respectively. Membership functions for x_1 and x_2 is shown in Fig.1.

Step 2: Solve the EVP in (26) using the LMI optimization toolbox in Matlab[15]. In this case, we obtain

$\varepsilon = 115.4292$, $\overline{W} = \begin{bmatrix} 0.0392 & -0.0236 \\ -0.0236 & 0.0464 \end{bmatrix}$,

$\overline{Q} = \begin{bmatrix} 62.41 & 16.19 & -36.87 & -18.78 \\ 16.19 & 45.86 & -18.78 & -31.12 \\ -36.87 & -18.78 & 36.87 & 18.78 \\ -18.78 & -31.12 & 18.78 & 31.12 \end{bmatrix}$ and the fuzzy estimation gains are found to be

$K_1 = \begin{bmatrix} 0.176 \\ -0.162 \end{bmatrix}$, $K_2 = \begin{bmatrix} 0.126 \\ -0.167 \end{bmatrix}$, $K_3 = \begin{bmatrix} 0.184 \\ -0.112 \end{bmatrix}$,

$K_4 = \begin{bmatrix} 0.396 \\ -0.473 \end{bmatrix}$, $K_5 = \begin{bmatrix} 0.314 \\ -0.475 \end{bmatrix}$, $K_6 = \begin{bmatrix} 0.348 \\ -0.329 \end{bmatrix}$,

$K_7 = \begin{bmatrix} 0.401 \\ -0.575 \end{bmatrix}$, $K_8 = \begin{bmatrix} 0.354 \\ -0.556 \end{bmatrix}$, $K_9 = \begin{bmatrix} 0.381 \\ -0.478 \end{bmatrix}$.

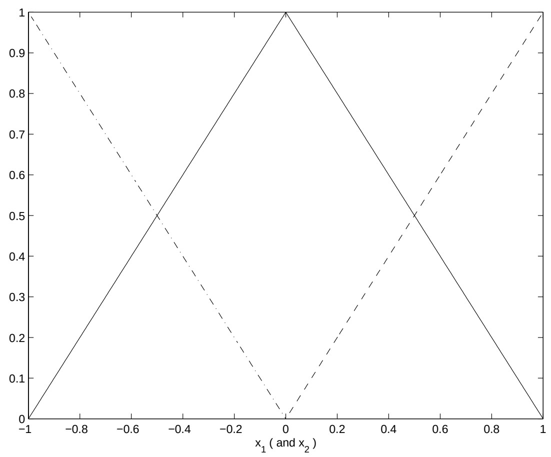

Figure 1. Membership function for x_1 and x_2.

The initial condition in the simulation is assumed to be $(x_1(0), x_2(0), \hat{x}_1(0), \hat{x}_2(0),)^T = (0.1, -0.1, 0, 0)^T$. Figs. 2-3 show the estimation errors (squared) for $x_1(\kappa) - \hat{x}_1(\kappa)$ and $x_2(\kappa) - \hat{x}_2(\kappa)$ using the proposed fuzzy filter and the extended Kalman filter (EKF).

The average squared errors $(\frac{1}{\hat{n}} \sum_{\hat{i}=1}^{\hat{n}} \{e^T(\hat{i}) e(\hat{i})\} \approx E\{e^T(\kappa) e(\kappa)\})$ for the proposed fuzzy filter and the extended Kalman filter are 0.0238 and 0.0616 (500 Monte Carlo runs, 100 samples each), respectively.

Remark 3: The uncertainties are not considered in the design of the extended Kalman filter. Therefore, in the simulation of this study, the EKF is constructed based on nominal system only, where the uncertainties are not considered.

4. Conclusions

In this paper, based on a Takagi and Sugeno fuzzy model, the suboptimal fuzzy estimation problems for nonlinear discrete-time uncertain stochastic systems are studied. Using a suboptimal approach, the outcome of the fuzzy estimation problems in this study is characterized in terms of an eigenvalue problem (EVP) by minimizing the upper bound on the variance of the estimation error. This EVP can be efficiently solved by convex optimization techniques.

This study extends the filtering design from linear uncertain stochastic systems to a class of nonlinear uncertain stochastic systems using fuzzy techniques. MI-based design procedure for the suboptimal fuzzy estimation problems of the nonlinear discrete-time uncertain stochastic systems is developed systematically. The proposed design procedure is very simple and can be performed efficiently using the LMI optimization toolbox in Matlab. Simulation example is given to illustrate the design procedure. Therefore, the proposed method is very suitable for practical applications.

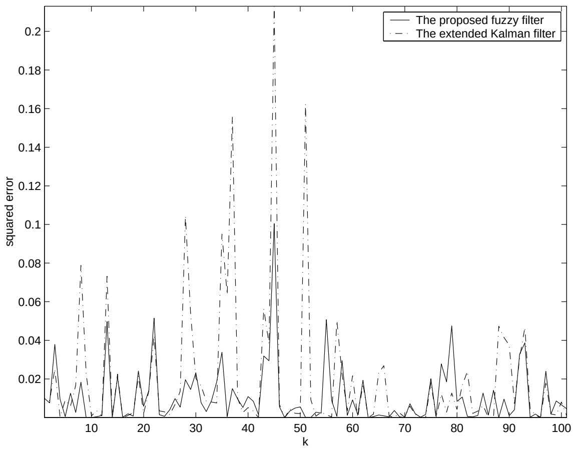

Figure 2.Estimation error (squared) of $[x_1(\kappa) - \hat{x}_1(\kappa)]^2$ (the proposed method (solid line) and EKF (dashdot line)).

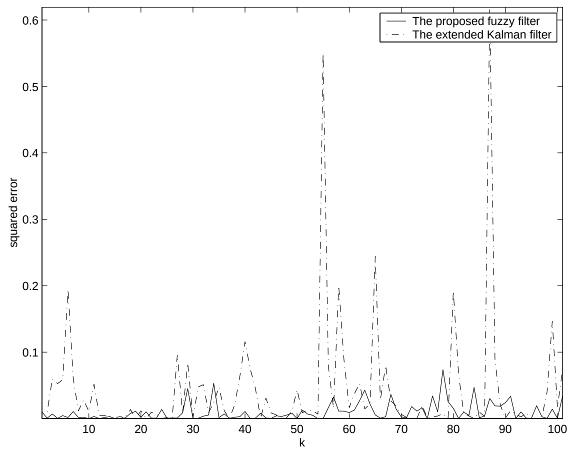

Figure 3. Estimation error (squared) of $[x_2(\kappa) - \hat{x}_2(\kappa)]^2$ (the proposed method (solid line) and EKF (dashdot line)).

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6. Appendix

$$A_1 = A_2 = A_3 = \begin{bmatrix} 0.3 & -0.2 \\ 0.2 & 0.3 \end{bmatrix}, \quad C_1 = [0.05 \quad 0.05],$$

$$A_4 = A_5 = A_6 = \begin{bmatrix} 0.5 & -0.2 \\ -0.1 & 0.3 \end{bmatrix}, \quad C_2 = [0.05 \quad 0.15],$$

$$A_7 = A_8 = A_9 = \begin{bmatrix} 0.1 & -0.2 \\ 0.5 & 0.3 \end{bmatrix}, \quad C_3 = [0.05 \quad -0.05],$$

$$U_{11} = \begin{bmatrix} 0.002 & 0.028 \\ 0.038 & 0.032 \end{bmatrix}, \quad C_4 = [0.15 \quad 0.05],$$

$$U_{12} = \begin{bmatrix} 0.004 & 0.026 \\ 0.036 & 0.034 \end{bmatrix}, \quad C_5 = [0.15 \quad 0.15],$$

$$U_{13} = \begin{bmatrix} 0.006 & 0.024 \\ 0.034 & 0.036 \end{bmatrix}, \quad C_6 = [0.15 \quad -0.05],$$

$$U_{14} = \begin{bmatrix} 0.008 & 0.022 \\ 0.032 & 0.038 \end{bmatrix}, \quad C_7 = [-0.05 \quad 0.05],$$

$$U_{15} = \begin{bmatrix} 0.01 & 0.02 \\ 0.03 & 0.04 \end{bmatrix}, \quad C_8 = [-0.05 \quad 0.15],$$

$$U_{16} = \begin{bmatrix} 0.012 & 0.018 \\ 0.028 & 0.042 \end{bmatrix}, \quad C_9 = [-0.05 \quad -0.05],$$

$$U_{17} = \begin{bmatrix} 0.014 & 0.016 \\ 0.026 & 0.044 \end{bmatrix}, \quad U_{18} = \begin{bmatrix} 0.016 & 0.014 \\ 0.024 & 0.046 \end{bmatrix},$$

$$U_{19} = \begin{bmatrix} 0.018 & 0.012 \\ 0.022 & 0.048 \end{bmatrix}, \quad V_{11} = V_{19} = \begin{bmatrix} 0.0015 & 0.001 \\ 0.009 & 0.01 \end{bmatrix},$$

$$V_{12} = V_{18} = \begin{bmatrix} 0.003 & 0.003 \\ 0.007 & 0.008 \end{bmatrix}, V_{13} = V_{17} = \begin{bmatrix} 0.0045 & 0.005 \\ 0.005 & 0.006 \end{bmatrix},$$

$$V_{14} = V_{16} = \begin{bmatrix} 0.006 & 0.007 \\ 0.003 & 0.004 \end{bmatrix}, V_{15} = \begin{bmatrix} 0.0075 & 0.009 \\ 0.001 & 0.002 \end{bmatrix},$$

$$V_{21} = V_{29} = \begin{bmatrix} 0.02 & 0.09 \\ 0.015 & 0.1 \end{bmatrix}, \quad U_{21} = [0.02 \quad 0.28],$$

$$V_{22} = V_{28} = \begin{bmatrix} 0.04 & 0.07 \\ 0.03 & 0.1 \end{bmatrix}, \quad U_{22} = [0.04 \quad 0.26],$$

$$V_{23} = V_{27} = \begin{bmatrix} 0.06 & 0.05 \\ 0.045 & 0.1 \end{bmatrix}, \quad U_{23} = [0.06 \quad 0.24],$$

$$V_{24} = V_{26} = \begin{bmatrix} 0.08 & 0.03 \\ 0.06 & 0.1 \end{bmatrix}, \quad U_{24} = [0.08 \quad 0.22],$$

$$V_{25} = \begin{bmatrix} 0.1 & 0.01 \\ 0.075 & 0.1 \end{bmatrix}, \quad U_{25} = [0.1 \quad 0.2],$$

$$U_{26} = [0.12 \quad 0.18], \quad U_{27} = [0.14 \quad 0.16],$$

$$U_{28} = [0.16 \quad 0.14], \quad U_{29} = [0.18 \quad 0.12].$$